Synchronized Formation Rotation and Attitude Control of Multiple Free-Flying Spacecraft

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In the observation slewing of long baseline interferometers formed by multiple free-flying spacecraft in formation, it is required to rotate the entire formation about a given axis and to synchronize individual spacecraft rotation with formation rotation. Using a particle model for spacecraft formation dynamics and a rigid-body model for spacecraft attitude dynamics, control laws are derived for this mode of operation in the absence of a gravitational field and disturbances. A simplified control law suitable for implementation is also obtained. It is shown that under mild conditions the formation alignment error decays to zero exponentially with time. Computer simulation studies are made for a free-flying spacecraft triad in a triangular formation. The results show that the developed control laws are effective in synchronized formation rotation.

Introduction

RECENT interest in developing long baseline interferometers using multiple spacecraft led to the study of various problems in the coordination and control of multiple spacecraft in formation. The use of multiple spacecraft for Michelson stellar interferometry was first proposed by Stachnik et al.[1,2] The feasibility of this approach for spacecraft in geosynchronous and low Earth orbits was studied by Johnson and Nock[3] and DeCue.[4]

The operation of such interferometers calls for observation slewing, which requires the rotation of the entire formation about a given axis. Moreover, each spacecraft must rotate in synchronism with formation rotation. In this paper, we consider multiple free-flying spacecraft in formation, which are of basic importance in long baseline deep space interferometers.[5] Attention is focused on developing control laws for formation rotation and attitude synchronization.

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We begin with the basic dynamic model of the multiple spacecraft to be used in the development of control laws for formation flying. This is followed by a discussion of the formation rotation and attitude synchronization problem. Then control laws for performing the required task are derived. The paper concludes with typical results of a computer simulation study for a free-flying spacecraft triad.

Dynamic Model for Formation Flying

We assume that there are N spacecraft to be flown in formation in the three-dimensional euclidean space R^3 and that each spacecraft is a rigid body with fixed center of mass. Let $\mathcal{F}_o$ denote the inertial frame with origin $O \in R^3$ and $\mathcal{F}_i$ a moving body frame whose origin O_i is at the mass center of the ith spacecraft (see Fig. 1). For a given orthonormal basis $\mathcal{B}_i$ for $\mathcal{F}_i$, the representation of a vector $\boldsymbol{a} \in R^3$ with respect to $\mathcal{B}_i$ will be denoted by $[\boldsymbol{a}]^i$. Let $\boldsymbol{r}_i(t)$ denote the position of the mass center of the ith spacecraft at time t in R^3 relative to $\mathcal{F}_o$. In the absence of a gravitational field and disturbances, the evolution of $\boldsymbol{r}_i(t)$ with time is governed by

$$\mathcal{D}_i^2(\boldsymbol{r}_i) = \boldsymbol{f}_{ci}/M_i \tag{1}$$

where $\boldsymbol{f}_{ci}$ and M_i denote the control thrust and the mass of the ith spacecraft, respectively; $\mathcal{D}_i^2$ is the time derivative operator defined by

$$\mathcal{D}_i^2 \triangleq \frac{{}^i\mathrm{d}^2(\cdot)}{\mathrm{d}t^2} + \frac{{}^i\mathrm{d}\boldsymbol{\omega}_i}{\mathrm{d}t} \times (\cdot) + 2\boldsymbol{\omega}_i \times \frac{{}^i\mathrm{d}(\cdot)}{\mathrm{d}t} + \boldsymbol{\omega}_i \times (\boldsymbol{\omega}_i \times \cdot) \tag{2}$$

where ${}^i\mathrm{d}/\mathrm{d}t$ and ${}^i(\mathrm{d}^2/\mathrm{d}t^2)$ denote the time derivative operators with respect to the body frame $\mathcal{F}_i$, and $\boldsymbol{\omega}_i$ is the angular velocity of $\mathcal{F}_i$ with respect to the inertial frame $\mathcal{F}_o$.

In formation flying, it is important to consider the relative motion between any pair of spacecraft. Let $\boldsymbol{\rho}_{ji} = \boldsymbol{r}_j - \boldsymbol{r}_i$ denote the position of the jth spacecraft relative to $\mathcal{F}_i$. Using Eqs. (1) and (2), it can be verified that the evolution of $\boldsymbol{\rho}_{ji}(t)$ with time is governed by

$$\mathcal{D}_i^2(\boldsymbol{\rho}_{ji}) = \boldsymbol{f}_{cj}/M_j - \boldsymbol{f}_{ci}/M_i \tag{3}$$

The attitude and angular velocity of the ith spacecraft with respect to the inertial frame $\mathcal{F}_o$ can be described by the following quaternion equations[6]

$$\frac{{}^o\mathrm{d}\hat{\boldsymbol{q}}_i}{\mathrm{d}t} = \frac{q_{i4}\boldsymbol{\omega}_i - \boldsymbol{\omega}_i \times \hat{\boldsymbol{q}}_i}{2}, \qquad \frac{\mathrm{d}q_{i4}}{\mathrm{d}t} = -\frac{\boldsymbol{\omega}_i \cdot \hat{\boldsymbol{q}}_i}{2} \tag{4}$$

and the Euler equation

$$\frac{{}^o\mathrm{d}(\boldsymbol{I}_i\boldsymbol{\omega}_i)}{\mathrm{d}t} = \boldsymbol{I}_i\frac{{}^i\mathrm{d}\boldsymbol{\omega}_i}{\mathrm{d}t} + \boldsymbol{\omega}_i \times (\boldsymbol{I}_i\boldsymbol{\omega}_i) = \boldsymbol{\tau}_{ci} \tag{5}$$

where the centered dot denotes the scalar product in R^3; $\boldsymbol{q}_i = (\hat{\boldsymbol{q}}_i^T, q_{i4})^T$ denotes the unit quaternion with q_{ij} being the Euler symmetric parameters; $\hat{\boldsymbol{q}}_i = (q_{i1}, q_{i2}, q_{i3})^T$; and $\boldsymbol{I}_i$ and $\boldsymbol{\tau}_{ci}$ are the tensor of inertia in the body frame $\mathcal{F}_i$ and control torque associated with the ith spacecraft, respectively.

Formation Rotation and Attitude Synchronization

There are many ways to fly spacecraft in formation. In the approach of Wang and Hadaegh,[7,8] the first spacecraft is taken as the reference spacecraft whose motion $\boldsymbol{r}_1 = \boldsymbol{r}_1(t)$, $t \geq 0$, serves as the reference motion for the remaining $N-1$ spacecraft (see Fig. 1). The desired motion for the ith spacecraft, $i = 2, \ldots, N$, is specified by

$$\boldsymbol{d}_i(t) = \boldsymbol{r}_1(t) + \boldsymbol{h}_i(t), \qquad t \geq 0 \tag{6}$$

where $\boldsymbol{h}_i(t) \in R^3$ is a specified nonzero deviation vector defined for all $t \geq 0$. The ith spacecraft tries to track the motion of the reference spacecraft such that the norm of the tracking error

$$\boldsymbol{E}_i(t) \triangleq \boldsymbol{d}_i(t) - \boldsymbol{r}_i(t) = \boldsymbol{\rho}_{1i}(t) + \boldsymbol{h}_i(t) \tag{7}$$

is as small as possible. The desired formation pattern at any time $t \geq 0$ is specified by the point set $\mathcal{P}(t) = \{\boldsymbol{r}_1(t), \boldsymbol{r}_1(t) + \boldsymbol{h}_2(t), \ldots, \boldsymbol{r}_1(t) + \boldsymbol{h}_N(t)\}$.

Another approach is based on nearest-neighbor tracking. Here the motion of the $(i-1)$th spacecraft serves as the reference motion for the ith spacecraft, and the desired motion for the ith spacecraft is specified by $\boldsymbol{d}_i(t) = \boldsymbol{r}_{i-1}(t) + \boldsymbol{h}_i(t)$, $t \geq 0$. This approach could lead to an undesirable formation pattern instability or oscillations. In what follows, we shall consider only the former case.

Now consider the formation body $\mathcal{C}(t)$ defined by the convex hull of $\mathcal{P}(t)$ [i.e., the set of points formed by all convex combinations of the points in $\mathcal{P}(t)$]. Let $\boldsymbol{R}(t)$ be a specified nonzero vector in R^3. It is required to rotate $\mathcal{C}(t)$ about an axis defined by the line $\mathcal{L}(t) = \boldsymbol{a}(t) + \mathrm{span}\{\boldsymbol{R}(t)\}$ with specified angular velocity $\boldsymbol{\Omega}(t)$ as illustrated in Fig. 2, where $\boldsymbol{a}(t)$ is a specified vector in R^3. Moreover, the reference spacecraft must rotate about a given body axis in synchronism with the formation body rotation, and the remaining spacecraft must track the reference spacecraft's attitude and angular
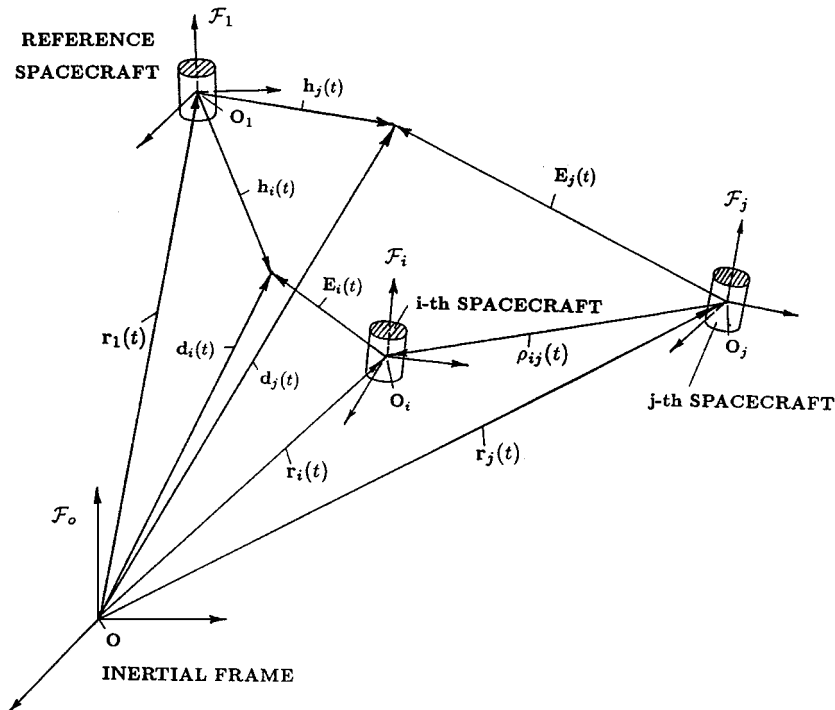

Fig. 1 Sketch of coordinate systems.

velocity. This implies that the desired angular speed of the reference spacecraft is $\|\mathbf{\Omega}(t) + \mathbf{\Omega}_o(t)\|$, and the desired angular velocity $\boldsymbol{\omega}_i^d(t)$ and unit quaternion $\boldsymbol{q}_i^d(t)$ for the ith spacecraft, $i = 2, \ldots, N$, are $\boldsymbol{\omega}_1(t)$ and $\boldsymbol{q}_1(t)$, respectively, where $\mathbf{\Omega}_o(t)$ is the angular velocity of $\mathcal{L}(t)$ with respect to $\mathcal{F}_o$. Note that $\boldsymbol{\omega}_i^d(t)$ and $\boldsymbol{q}_i^d(t)$ must be compatible with each other in the sense that they satisfy

$$\frac{{}^o\mathrm{d}\hat{\boldsymbol{q}}_i^d}{\mathrm{d}t} = \frac{q_{i4}^d \boldsymbol{\omega}_i^d - \boldsymbol{\omega}_i^d \times \hat{\boldsymbol{q}}_i^d}{2}, \qquad \frac{\mathrm{d}q_{i4}^d}{\mathrm{d}t} = -\frac{\boldsymbol{\omega}_i^d \cdot \hat{\boldsymbol{q}}_i^d}{2} \qquad (8a)$$
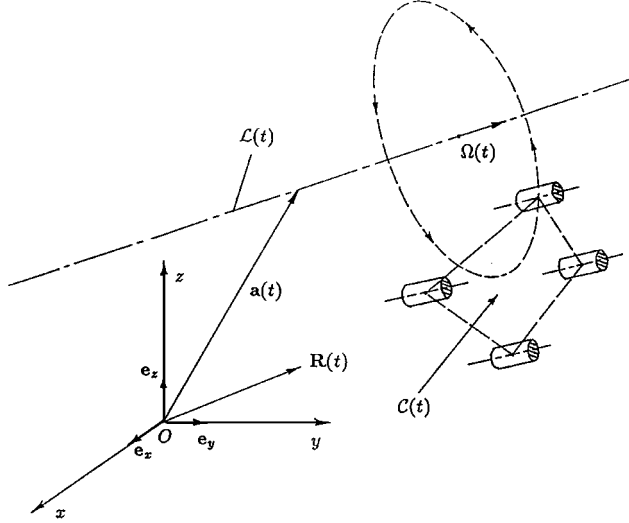
Fig. 2 Formation rotation about the line $\mathcal{L}(t)$.

and

$$\frac{{}^o\mathrm{d}\left(I_i \boldsymbol{\omega}_i^d\right)}{\mathrm{d}t} = I_i \frac{{}^i\mathrm{d}\boldsymbol{\omega}_i^d}{\mathrm{d}t} + \boldsymbol{\omega}_i^d \times \left(I_i \boldsymbol{\omega}_i^d\right) \triangleq \boldsymbol{\tau}_{ci}^d \qquad (8b)$$

where $\boldsymbol{q}_i^d = [(\hat{\boldsymbol{q}}_i^d)^T, q_{i4}^d]^T$.

To clarify the foregoing notion of formation rotation and attitude synchronization, we consider a spacecraft triad flying in a triangular formation as shown in Fig. 3. Let $[\boldsymbol{r}]^o = (x, y, z)^T$ denote the representation of a point $\boldsymbol{r}$ with respect to an orthonormal basis $\mathcal{B}_o = \{\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z\}$ in the inertial frame $\mathcal{F}_o$. We assume that the reference spacecraft moves along a circular orbit $\mathcal{O} = \{(x, y, z): x^2 + y^2 = a_o^2, z = z_o\}$ with radius a_o and constant angular velocity $-\omega_o \boldsymbol{e}_z$, and the position of the reference spacecraft is given by

$$\boldsymbol{r}_1(t) = [a_o \cos(\omega_o t)]\boldsymbol{e}_x - [a_o \sin(\omega_o t)]\boldsymbol{e}_y + z_o \boldsymbol{e}_z \qquad (9)$$

To define the desired initial formation, we introduce constant deviation vectors $\boldsymbol{h}_2^o$ and $\boldsymbol{h}_3^o$:

$$\boldsymbol{h}_2^o = h_{2x}^o \boldsymbol{e}_x + h_{2y}^o \boldsymbol{e}_y + h_{2z}^o \boldsymbol{e}_z$$
$$\boldsymbol{h}_3^o = -h_{2x}^o \boldsymbol{e}_x - h_{2y}^o \boldsymbol{e}_y + h_{2z}^o \boldsymbol{e}_z \qquad (10a)$$

Then the desired initial formation pattern at time t is specified by the point set

$$\mathcal{P}(t) = \left\{ \boldsymbol{r}_1(t), \boldsymbol{r}_1(t) + \boldsymbol{h}_2^o, \boldsymbol{r}_1(t) + \boldsymbol{h}_3^o \right\} \qquad (10b)$$

and the formation body $\mathcal{C}(t)$ is the plane domain bounded by the isosceles triangle with vertices given by $\mathcal{P}(t)$. For a space interferometer, the combiner and collectors correspond to the reference and the remaining spacecraft, respectively.[1-5] We assume that the spacecraft body is a rigid cylinder with uniform mass density. Let $\mathcal{B}_i = \{\boldsymbol{e}_{x'}^i, \boldsymbol{e}_{y'}^i, \boldsymbol{e}_{z'}^i\}$ be an orthonormal basis for the body frame $\mathcal{F}_i$
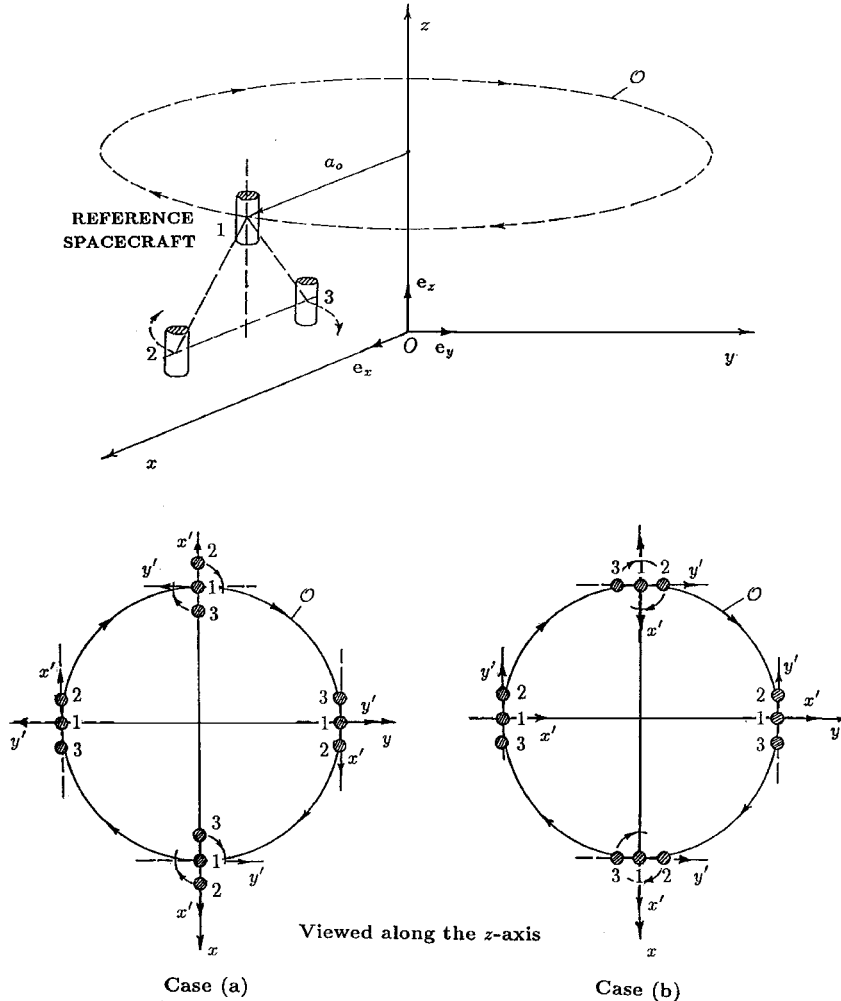

Fig. 3 Rotation of spacecraft triad in a triangular formation.

for the ith spacecraft such that $e_{z'}^i$ is along the cylinder axis that is also aligned with e_z. We consider two different cases: a) the x' axis is aligned with x axis, and b) the y' axis passes through the center of the circular orbit $\mathcal{O}$ at all times, except during formation rotation (see Fig. 3). Case a corresponds to the situation where it is required to have the instruments or solar panels on board the spacecraft pointing to a fixed direction. Thus, the desired angular velocities ω_1^d for cases a and b are zero and $-\omega_o e_z$, respectively.

Now, suppose that the objective is to rotate the triangular formation about the reference spacecraft's cylinderical body axis with constant angular velocity $\Omega = -\omega_R e_z$. Thus, the rotation axis $\mathcal{L}(t)$ for $\mathcal{C}(t)$ is simply span$\{e_z\}$, and the deviation vectors $h_2(t)$ and $h_3(t)$ take on the following forms:

$$h_2(t) = \left[h_{2x}^o \cos(\omega_R t) - h_{2y}^o \sin(\omega_R t) \right] e_x$$
$$+ \left[h_{2x}^o \sin(\omega_R t) + h_{2y}^o \cos(\omega_R t) \right] e_y + h_{2z}^o e_z$$
$$\tag{11}$$
$$h_3(t) = \left[-h_{2x}^o \cos(\omega_R t) + h_{2y}^o \sin(\omega_R t) \right] e_x$$
$$+ \left[-h_{2x}^o \sin(\omega_R t) - h_{2y}^o \cos(\omega_R t) \right] e_y + h_{2z}^o e_z$$

During formation rotation, the desired angular velocity ω_1^d for the reference spacecraft relative to the inertial frame $\mathcal{F}_o$ is $-\omega_R e_z$ for case a and $-(\omega_o + \omega_R) e_z$ for case b. The corresponding desired quaternion for the reference spacecraft can be determined from Eq. (8a), which can be written explicitly as

$$\frac{{}^o\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} q_{11}^d \\ q_{12}^d \\ q_{13}^d \\ q_{14}^d \end{bmatrix} = \begin{bmatrix} 0 & \omega_T & 0 & 0 \\ -\omega_T & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_T \\ 0 & 0 & -\omega_T & 0 \end{bmatrix} \begin{bmatrix} q_{11}^d \\ q_{12}^d \\ q_{13}^d \\ q_{14}^d \end{bmatrix} \tag{12}$$

where $\omega_T = \omega_R/2$ for case a and $\omega_T = (\omega_o + \omega_R)/2$ for case b.

The solution to Eq. (12) with initial condition $[\hat{q}_1(t_1)^T, q_{14}(t_1)]^T$ at time t_1 (the starting time for formation rotation) is given by

$$\begin{bmatrix} q_{11}^d(t) \\ q_{12}^d(t) \end{bmatrix} = \Phi(t - t_1) \begin{bmatrix} q_{11}(t_1) \\ q_{12}(t_1) \end{bmatrix}$$
$$\tag{13}$$
$$\begin{bmatrix} q_{13}^d(t) \\ q_{14}^d(t) \end{bmatrix} = \Phi(t - t_1) \begin{bmatrix} q_{13}(t_1) \\ q_{14}(t_1) \end{bmatrix}, \qquad t \geq t_1$$

where

$$\Phi(t - t_1) = \begin{bmatrix} \cos[\omega_T(t - t_1)] & \sin[\omega_T(t - t_1)] \\ -\sin[\omega_T(t - t_1)] & \cos[\omega_T(t - t_1)] \end{bmatrix} \tag{14}$$

Control Laws

To derive a control law for formation acquisition and rotation, we consider the following equation for the tracking error $E_i(t)$, $i = 2, \ldots, N$, defined by Eq. (7):

$$\mathcal{D}_i^2(E_i) \triangleq \ddot{E}_i + \dot{\omega}_i \times E_i + 2\omega_i \times \dot{E}_i + \omega_i \times (\omega_i \times E_i)$$
$$= \mathcal{D}_i^2(h_i) + (f_{c1}/M_1) - (f_{ci}/M_i) \tag{15}$$

and the following positive definite function of $(E_i, \dot{E}_i)$ defined on R^6:

$$V_{1i} = \frac{K_{1i} E_i \cdot E_i + \dot{E}_i \dot{E}_i}{2}, \qquad K_{1i} > 0 \tag{16}$$

where $\dot{E}_i \triangleq {}^i dE_i/dt$, $\ddot{E}_i \triangleq {}^i d^2 E_i/dt^2$, and $\dot{\omega}_i \triangleq {}^i d\omega_i/dt$. The time rate of change of V_{1i} along any solution of Eq. (15) is given by

$$\frac{\mathrm{d}V_{1i}}{\mathrm{d}t} = \dot{E}_i \cdot \left\{ \mathcal{D}_i^2(h_i) + \frac{f_{c1}}{M_1} - 2\omega_i \times \dot{E}_i + I_i^{-1}[\omega_i \times (I_i \omega_i) - \tau_{ci}] \right.$$
$$\left. \times E_i - \omega_i \times (\omega_i \times E_i) + K_{1i} E_i - \frac{f_{ci}}{M_i} \right\} = \dot{E}_i \cdot \left(w_i - \frac{f_{ci}}{M_i} \right) \tag{17}$$

where

$$w_i = f_{c1}/M_1 + I_i^{-1}[\omega_i \times (I_i \omega_i) - \tau_{ci}] \times E_i$$
$$- (\omega_i \cdot E_i)\omega_i + \left(K_{1i} + \|\omega_i\|^2 \right) E_i + \mathcal{D}_i^2(h_i) \tag{18}$$

If we set

$$f_{ci}/M_i = w_i + K_{2i}\dot{E}_i \tag{19}$$

where K_{2i} is a positive constant, then $dV_{i1}/dt = -K_{2i}\|\dot{E}_i\|^2 \leq 0$, and Eq. (15) reduces to

$$\ddot{E}_i(t) + K_{2i}\dot{E}_i(t) + 2\omega_i(t) \times \dot{E}_i(t) + K_{1i}E_i(t) = 0 \tag{20}$$

Because $\dot{E}_i(t) \cdot [\omega_i(t) \times \dot{E}_i(t)] = 0$ for all $\omega_i(t)$, it follows that all solutions $[E_i(t), \dot{E}_i(t)]$ of Eq. (20) tend to $(0, 0) \in R^6$ as $t \to \infty$ for any $K_{1i}, K_{2i} > 0, i = 2, \ldots, N$. Moreover, the zero state of Eq. (20) is totally stable or stable under persistent disturbances.[7,9] This property implies that asymptotic stability of the zero state of Eq. (20) is preserved in the presence of small state-dependent perturbations.

In the case where the components of the control thrusts f_{ci} are amplitude limited, i.e., $[f_{ci}]^i = (f_{ci1}, f_{ci2}, f_{ci3})^T$ satisfies $|f_{cij}| \leq F_{ci}, j = 1, 2, 3$, where $F_{ci}, i = 2, \ldots, N$, are given positive constants, we consider again Eq. (17) rewritten in reference to a given orthonormal basis $\mathcal{B}_i$ for $\mathcal{F}_i$:

$$\frac{\mathrm{d}V_{1i}}{\mathrm{d}t} = \sum_{j=1}^{3} \left\{ [\dot{E}_i]_j^i \left([w_i]_j^i - \frac{f_{cij}}{M_i} \right) \right\} \tag{17'}$$

where $[w_i]_j^i$ denotes the jth component of $[w_i]^i$. If we set

$$f_{cij} = F_{ci} \, \mathrm{sat}\left[g_{ij}\left(\omega_i, [E_i]_j^i, [\dot{E}_i]_j^i, f_{c1}, h_i \right) \right]$$
$$j = 1, 2, 3 \tag{21}$$

where sat$(\cdot)$ denotes the saturation function defined by sat$(a) = $ sign(a) if $|a| \geq 1$ and sat$(a) = a$ if $|a| \leq 1$, and

$$g_{ij}\left(\omega_i, [E_i]_j^i, [\dot{E}_i]_j^i, f_{c1}, h_i \right) \triangleq (M_i/F_{ci})\left([w_i]_j^i + K_{2i}[\dot{E}_i]_j^i \right)$$
$$j = 1, 2, 3 \tag{22}$$

then

$$\frac{\mathrm{d}V_{1i}}{\mathrm{d}t} = \sum_{j=1}^{3} \left\{ [\dot{E}_i]_j^i \left([w_i]_j^i \right. \right.$$
$$\left. \left. - \frac{F_{ci}}{M_i} \mathrm{sat}\left[g_{ij}\left(\omega_i, [E_i]_j^i, [\dot{E}_i]_j^i, f_{c1}, h_i \right) \right] \right) \right\} \tag{23}$$

For control law (21), Eq. (15) has the following representation with respect to basis $\mathcal{B}_i$:

$$\left[\mathcal{D}_i^2(E_i) \right]_j^i = \left[\mathcal{D}_i^2(h_i) + f_{c1}/M_1 \right]_j^i$$
$$- (F_{ci}/M_i) \, \mathrm{sat}\left[g_{ij}\left(\omega_i, [E_i]_j^i, [\dot{E}_i]_j^i, f_{c1}, h_i \right) \right], \qquad j = 1, 2, 3 \tag{24}$$

It can be readily verified that if $\|\omega_i(t)\|$, $\|f_{c1}(t)\|$, $\|\mathcal{D}_i^2(h_i)(t)\|$, and $\|\tau_{ci}(t)\|$ are uniformly bounded for all $t \geq 0$, then the set $\{([E_i]^i, [\dot{E}_i]^i) \in R^6 : |g_{ij}(\omega_i, [E_i]_j^i, [\dot{E}_i]_j^i, f_{c1}, h_i)| \leq 1, j = 1, 2, 3\}$, contains a neighborhood of the zero state $(0, 0) \in R^6$. Consequently, any solution $\{[E_i(t), \dot{E}_i(t)]\}^i$ of Eq. (24) tends to the zero state as $t \to \infty$ for any feedback gains K_{1i} and $K_{2i} > 0$ and sufficiently small $\|\{[E_i(0)]^i, [\dot{E}_i(0)]^i\}\|$.

To derive control laws for the reference and remaining spacecraft to achieve attitude synchronization, we introduce

$$\delta q_i \triangleq q_i^d - q_i = \left[\left(\hat{q}_i^d - \hat{q}_i \right)^T, q_{i4}^d - q_{i4} \right]^T, \qquad \delta \omega_i \triangleq \omega_i^d - \omega_i \tag{25}$$

where $q_i^d = q_1$, and $\omega_i^d = \omega_1$, for $i = 2, \ldots, N$. It follows from Eqs. (4), (5), and (8) that δq_i and $\delta \omega_i$ satisfy

$$\frac{{}^o\mathrm{d}\delta \hat{\boldsymbol{q}}_i}{\mathrm{d}t} = \frac{q_{i4}^d \boldsymbol{\omega}_i^d - q_{i4}\boldsymbol{\omega}_i - \boldsymbol{\omega}_i^d \times \boldsymbol{q}_i^d + \boldsymbol{\omega}_i \times \hat{\boldsymbol{q}}_i}{2}$$

$$\frac{\mathrm{d}\delta q_{14}}{\mathrm{d}t} = -\frac{\boldsymbol{\omega}_i^d \cdot \hat{\boldsymbol{q}}_i^d - \boldsymbol{\omega}_i \cdot \hat{\boldsymbol{q}}_i}{2} \qquad (26)$$

and

$$\frac{{}^o\mathrm{d}(\boldsymbol{I}_i \delta \boldsymbol{\omega}_i)}{\mathrm{d}t} = \boldsymbol{I}_i \frac{{}^i\mathrm{d}\delta \boldsymbol{\omega}_i}{\mathrm{d}t} + \delta \boldsymbol{\omega}_i \times \boldsymbol{I}_i \boldsymbol{\omega}_i^d + \boldsymbol{\omega}_i^d \times \boldsymbol{I}_i \delta \boldsymbol{\omega}_i$$

$$- \delta \boldsymbol{\omega}_i \times \boldsymbol{I}_i \delta \boldsymbol{\omega}_i = \boldsymbol{\tau}_{ci}^d - \boldsymbol{\tau}_{ci} \qquad (27)$$

By requiring the time rate of change of the following positive definite function along any solution of Eqs. (26) and (27) to be nonpositive for all $t \geq 0$:

$$V_{2i} = K_{qi}\left(\delta q_{i4}^2 + \delta \hat{\boldsymbol{q}}_i \cdot \delta \hat{\boldsymbol{q}}_i\right) + \frac{\delta \boldsymbol{\omega}_i \cdot \boldsymbol{I}_i \delta \boldsymbol{\omega}_i}{2} \qquad (28)$$

we obtain the following attitude control law:

$$\boldsymbol{\tau}_{ci} = K_{qi}\boldsymbol{s} + K_{\omega i}\boldsymbol{I}_i \delta \boldsymbol{\omega}_i + \boldsymbol{\tau}_{ci}^d - \boldsymbol{\omega}_i^d \times \frac{\boldsymbol{I}_i \delta \boldsymbol{\omega}_i}{2} \qquad (29)$$

where K_{qi} and $K_{\omega i}$ are positive constant feedback gains, and

$$\boldsymbol{s} = q_{i4}^d \delta \hat{\boldsymbol{q}}_i - \delta q_{i4} \hat{\boldsymbol{q}}_i^d - \hat{\boldsymbol{q}}_i^d \times \delta \hat{\boldsymbol{q}}_i \qquad (30)$$

It can be verified[7] that under the action of control law (29), any solution $[\delta \hat{\boldsymbol{q}}_i(t), \delta q_{14}(t), \delta \boldsymbol{\omega}_i(t)]$ of Eqs. (26) and (27) tends to $(\mathbf{0}, 0, \mathbf{0}) \in R^7$ as $t \to \infty$.

In the case where the control torques $\boldsymbol{\tau}_{ci}$ are amplitude limited {i.e., $[\boldsymbol{\tau}_{ci}]^i = (\tau_{ci1}, \tau_{ci2}, \tau_{ci3})^T$ satisfies $|\tau_{cij}| \leq T_{ci}$, $j = 1, 2, 3$}, where T_{ci}, $i = 1, \ldots, N$, are given positive constants, we set

$$[\boldsymbol{\tau}_{ci}]_j^i = T_{ci} \operatorname{sat}\left\{\frac{\left[K_{qi}\boldsymbol{s} + K_{\omega i}\boldsymbol{I}_i \delta \boldsymbol{\omega}_i + \boldsymbol{\tau}_{ci}^d - \boldsymbol{\omega}_i^d \times (\boldsymbol{I}_i \delta \boldsymbol{\omega}_i)/2\right]_j^i}{T_{ci}}\right\}$$

$$j = 1, 2, 3 \qquad (31)$$

Here, because the control torques are amplitude limited, loss of synchronization can occur when the angular speed of formation rotation or the reference spacecraft is so high that the remaining spacecraft are unable to track the formation rotation. The possible occurrence of this phenomenon can be deduced from the following inequality obtained by first integrating Eq. (27) from time t_o to t, and making use of the triangle inequality $\|\boldsymbol{a} - \boldsymbol{b}\| \geq \|\boldsymbol{a}\| - \|\boldsymbol{b}\|$ for norms of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$:

$$\|\boldsymbol{I}_i[\delta \boldsymbol{\omega}_i(t) - \delta \boldsymbol{\omega}_i(t_o)]\|_\infty = \left\|\int_{t_o}^{t}\left[\boldsymbol{\tau}_{ci}^d(s) - \boldsymbol{\tau}_{ci}(s)\right]\mathrm{d}s\right\|_\infty$$

$$\geq \left\|\int_{t_o}^{t}\boldsymbol{\tau}_{ci}^d(s)\,\mathrm{d}s\right\|_\infty - \left\|\int_{t_o}^{t}\boldsymbol{\tau}_{ci}(s)\,\mathrm{d}s\right\|_\infty \geq \left\|\int_{t_o}^{t}\boldsymbol{\tau}_{ci}^d(s)\,\mathrm{d}s\right\|_\infty$$

$$- \int_{t_o}^{t}\|\boldsymbol{\tau}_{ci}(s)\|_\infty\,\mathrm{d}s \geq \left\|\int_{t_o}^{t}\boldsymbol{\tau}_{ci}^d(s)\,\mathrm{d}s\right\|_\infty - T_{ci}(t - t_o)$$

$$t \geq t_o \qquad (32)$$

where $\|\boldsymbol{a}\|_\infty \triangleq \max\{|a_i|; i = 1, 2, 3\}$ and $\boldsymbol{a} = (a_1, a_2, a_3)$. Now suppose that the desired angular speed is such that the various corresponding $\boldsymbol{\tau}_{ci}^d$ satisfy

$$\left\|\int_{t_o}^{t}\boldsymbol{\tau}_{ci}^d(s)\,\mathrm{d}s\right\|_\infty - T_{ci}(t - t_o) \geq \eta_i(t - t_o)$$

for all t in some time interval $I_T = [t_o, T]$ and some positive constant η_i. Then the norm of the deviation in the angular momentum of the ith spacecraft from its desired value grows with time over I_T. Consequently, loss of synchronization results.

From Eqs. (18) and (19), it is evident that the formation control law consists of feedback terms involving $\boldsymbol{E}_i(t)$, $\dot{\boldsymbol{E}}_i(t)$, $\boldsymbol{\omega}_i(t)$, and $\boldsymbol{\tau}_{ci}(t)$ and feedforward terms involving $\mathcal{D}_i^2(\boldsymbol{h}_i)(t)$ and the control thrust $\boldsymbol{f}_{c1}(t)$ of the reference spacecraft. Figure 4 shows the structure of the overall control system. The reference-path parameters are fed into the reference-path command generator, which produces the necessary data for generating the controls for the reference spacecraft. Similarly, the formation-pattern parameters are fed into the formation-command generator for the ith spacecraft, whose output along with the displacement, velocity, quaternion, and angular velocity of the ith spacecraft relative to the reference spacecraft are used in generating the controls for the ith spacecraft.
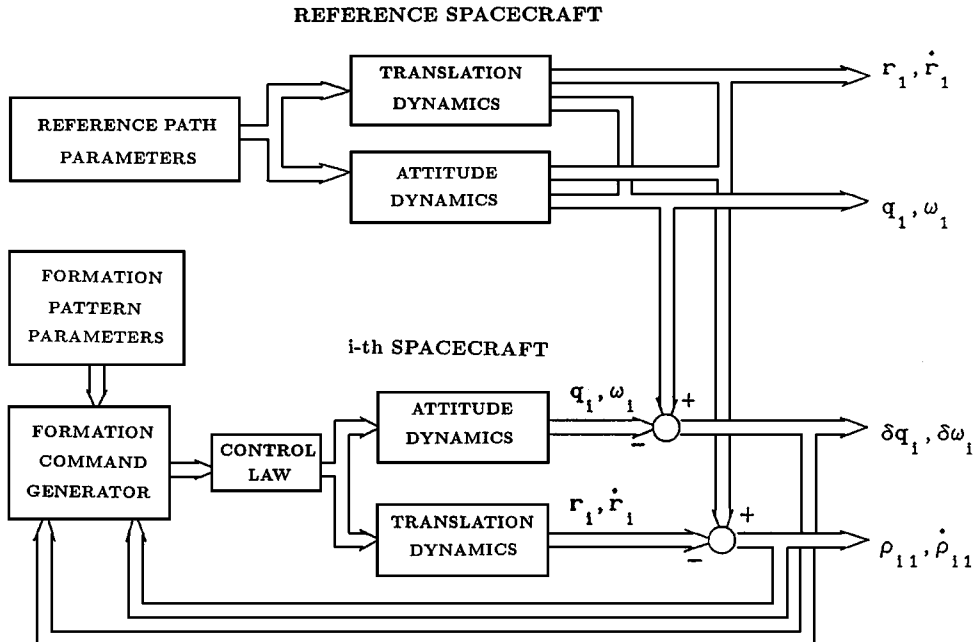

Fig. 4   Structure of the overall control system.
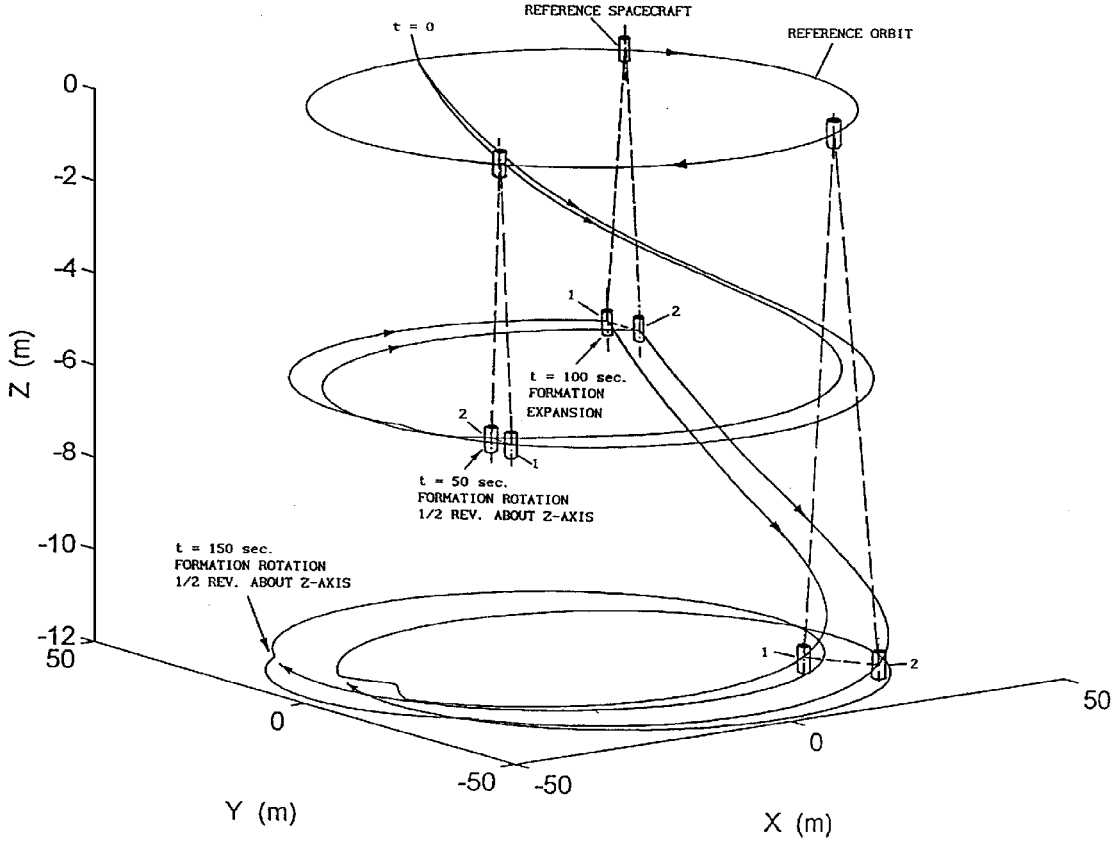
SPACECRAFT TRIAD MOTION IN 3D-SPACE

Fig. 5 Trajectories of a spacecraft triad undergoing formation reconfiguration and rotation in three-dimensional space.

Simplified Control Laws

Control laws (19) and (21) for formation acquisition and rotation have complex forms that cannot be readily implemented. In what follows, we shall consider a simplified version of control law (19) given by

$$f_{ci}/M_i = K_{1i}E_i + K_{2i}\dot{E}_i + \tilde{w}_i \tag{33}$$

where K_{1i} and K_{2i} are positive self-adjoint linear transformations on R^3 onto R^3, and

$$\tilde{w}_i = f_{c1}/M_1 + \mathcal{D}_i^2(h_i) \tag{34}$$

Substituting Eq. (33) into Eq. (15) leads to the following linear time-varying differential equation for the tracking error:

$$\ddot{E}_i(t) + R_i(t)\dot{E}_i(t) + S_i(t)E_i(t) = 0 \tag{35}$$

where $R_i(t)$ and $S_i(t)$ are linear transformations on R^3 into R^3 defined by

$$R_i(t) = K_{2i} + 2\omega_i(t) \times (\cdot) \tag{36}$$

and

$$S_i(t) = \dot{\omega}_i(t) \times (\cdot) + \omega_i(t)[\omega_i(t)\cdot] + \left[K_{1i} - \|\omega_i(t)\|^2 I\right] \tag{37}$$

Consider the following function defined on R^6:

$$\tilde{V}_i = \frac{E_i \cdot K_{1i}E_i + \dot{E}_i \cdot \dot{E}_i + 2\gamma \dot{E}_i \cdot E_i}{2} \tag{38}$$

where γ is a positive constant. Because K_{1i} is a positive self-adjoint linear transformation, hence

$$E_i \cdot K_{1i}E_i \geq \lambda_{\min}(K_{1i})\|E_i\|^2 \tag{39}$$

where $\lambda_{\min}(K_{1i})$ denotes the minimum eigenvalue of K_{1i}. From Eq. (39) and the inequality $\dot{E}_i \cdot E_i \geq -\|\dot{E}_i\|\|E_i\|$, it follows that $\tilde{V}_i$ satisfies the lower bound:

$$\tilde{V}_i \geq \frac{1}{2}\begin{bmatrix} \|E_i\| \\ \|\dot{E}_i\| \end{bmatrix}^T P \begin{bmatrix} \|E_i\| \\ \|\dot{E}_i\| \end{bmatrix} \geq \frac{1}{2}\lambda_{\min}(P)\|(E_i, \dot{E}_i)\|^2 \tag{40}$$

where $\|(E_i, \dot{E}_i)\|^2 = \|E_i\|^2 + \|\dot{E}_i\|^2$,

$$P = \begin{bmatrix} \lambda_{\min}(K_{1i}) & -\gamma \\ -\gamma & 1 \end{bmatrix} \tag{41}$$

Thus, if

$$\lambda_{\min}(K_{1i}) \geq \gamma^2 \tag{42}$$

then $\tilde{V}_i$ is positive definite on R^6. We shall show that under certain mild conditions the zero state of Eq. (35) is exponentially stable.

Consider the time rate of change of $\tilde{V}_i$ given by

$$\frac{d\tilde{V}_i}{dt} = \gamma E_i \cdot \left(\|\omega_i\|^2 I - K_{1i}\right)E_i + \dot{E}_i \cdot (\gamma I - K_{2i})\dot{E}_i$$

$$- \gamma(\omega_i \cdot E_i)^2 - \dot{E}_i \cdot (\dot{\omega}_i \times E_i) - (\omega_i \cdot E_i)(\dot{E}_i \cdot \omega_i)$$

$$+ E_i \cdot \left(\|\omega_i\|^2 I - \gamma K_{2i}\right)\dot{E}_i - 2\gamma E_i \cdot (\omega_i \times \dot{E}_i) \tag{43}$$

From Eq. (5), it is evident that for bounded control torque τ_{ci} [i.e., $\|\tau_{ci}(t)\| \leq \hat{\tau}_{ci} < \infty$ for all $t \geq 0$] there exist positive constants α_i and β_i such that

$$\sup\{\|\omega_i(t)\|; t \geq 0\} \leq \alpha_i < \infty$$

$$\sup\{\|\dot{\omega}_i(t)\|; t \geq 0\} \leq \beta_i < \infty \tag{44}$$

Then $d\tilde{V}_i/dt$ satisfies the following estimate:

$$\frac{d\tilde{V}_i}{dt} \leq -\begin{bmatrix} \|E_i\| \\ \|\dot{E}_i\| \end{bmatrix}^T Q \begin{bmatrix} \|E_i\| \\ \|\dot{E}_i\| \end{bmatrix} \tag{45}$$
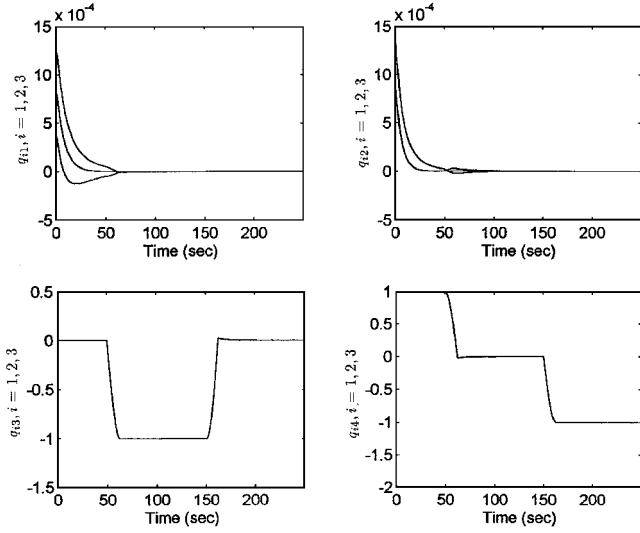
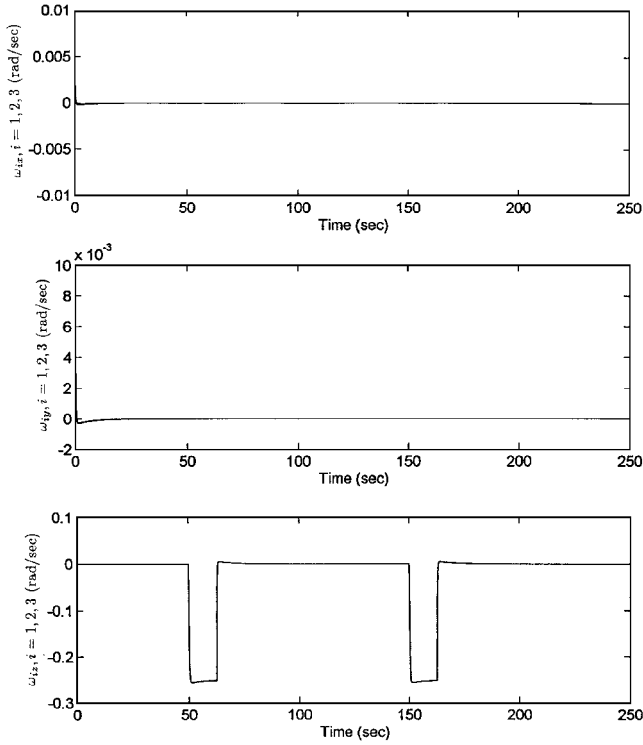Fig. 6a   Quaternion components of spacecraft triad vs time.

Fig. 6b   Angular velocity components of spacecraft triad vs time.

where

$$Q = \begin{Bmatrix} \gamma \left[\lambda_{\min}(K_{1i}) - \alpha_i^2 \right] & \delta_{1i} \\ \delta_{1i} & \lambda_{\min}(K_{2i}) - \gamma \end{Bmatrix} \quad (46)$$

where $\delta_{1i} = (\beta_i - 2\gamma\alpha_i + \alpha_i^2 + \gamma\|K_{2i}\|)/2$. Now if

$$\lambda_{\min}(K_{1i}) > \alpha_i^2, \qquad \lambda_{\min}(K_{2i}) > \gamma$$
$$\gamma\left[\lambda_{\min}(K_{1i}) - \alpha_i^2\right][\lambda_{\min}(K_{2i}) - \gamma] > \delta_{1i}^2 \quad (47)$$

then Q is positive definite. Thus, under conditions (42) and (47), we have

$$\frac{d\tilde{V}_i}{dt} \le -\lambda_{\min}(Q)\left(\|E_i\|^2 + \|\dot{E}_i\|^2\right) \le -2\left\{\frac{\lambda_{\min}(Q)}{\lambda_{\min}(P)}\right\}\tilde{V}_i \quad (48)$$

It follows that

$$\tilde{V}_i(t) \le \tilde{V}_i(0) \exp\left\{-2\left[\frac{\lambda_{\min}(Q)}{\lambda_{\min}(P)}\right]t\right\} \quad (49)$$
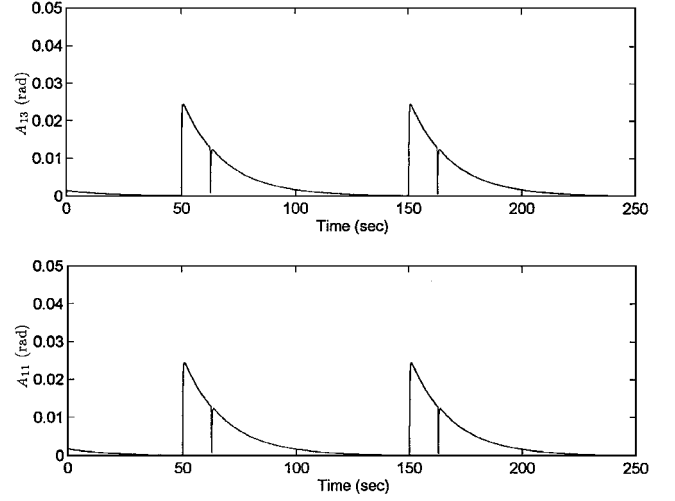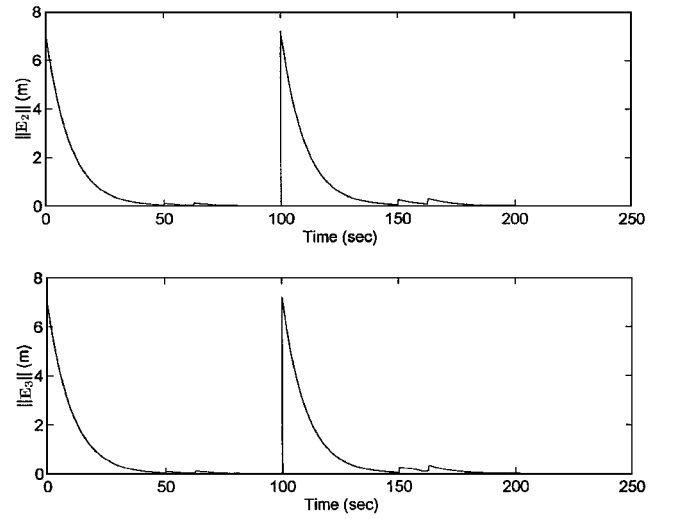

Fig. 7a   Time records for the attitude errors of spacecraft 2 and 3 relative to the reference spacecraft (A_{1i}—relative attitude angle between the ith and the reference spacecraft).

Fig. 7b   Time records for the norm of position tracking errors of spacecraft 2 and 3 relative to the reference spacecraft.

for all $t \ge 0$, which implies exponential stability of the zero state of Eq. (35).

Remark: The inclusion of the term $\tilde{w}_i$ in control law (33) is essential in achieving exponential stability. Because $\tilde{w}_i$ given by Eq. (34) involves the deviation vector $h_i(t)$ and control law f_{c1} associated with the reference spacecraft, these data must be transmitted to the ith spacecraft. If the ith spacecraft receives the deviation vector $h_i(t)$ and its velocity $\dot{h}_i(t)$ from the reference spacecraft and has onboard sensors to measure the relative position $\rho_{i1}(t)$ and velocity $\dot{\rho}_{i1}(t)$, then the positional error and error rate can be determined by $E_i(t) = h_i(t) - \rho_{i1}(t)$ and $\dot{E}_i(t) = \dot{h}_i(t) - \dot{\rho}_{i1}(t)$, respectively. These data are required for the implementation of the simplified control law (33).

Simulation Study

To determine the effectiveness of the proposed control laws, we consider a spacecraft triad discussed earlier. The spacecraft parameter values are given in the Appendix. We present only typical simulation results for case a where the desired angular velocity for the reference spacecraft is zero before and after formation rotation.

Consider the scenario where the first spacecraft generates a circular reference orbit for the remaining two spacecraft given by Eq. (9) with $\omega_o = 0.071$ rad/s. The spacecraft are initially clustered and expand to a triangular formation pattern specified by $\mathcal{P}(t)$ given by Eq. (10b) with

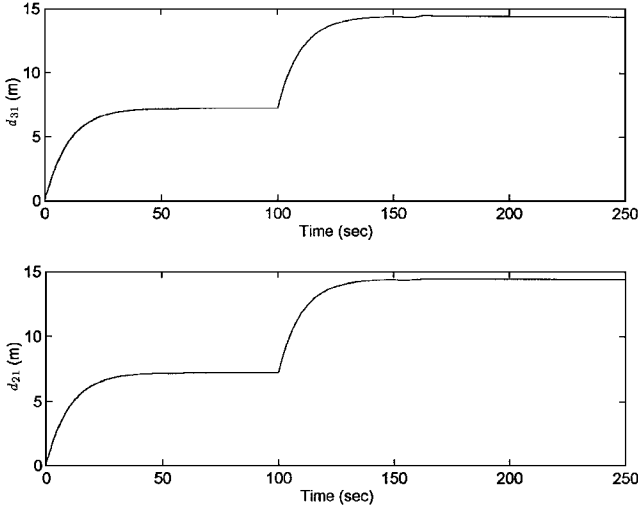$$h_2^o = 4e_y - 6e_z, \qquad h_3^o = -4e_y - 6e_z \quad (50)$$

Fig. 8. Time records for the distances between spacecraft 2, 3, and the reference spacecraft (d_{i1}—distance between the ith and the reference spacecraft).

At $t = 50$ s, the formation begins rotation for half a revolution about the vertical axis with desired constant angular velocity $\mathbf{\Omega} = -0.25\mathbf{e}_z$ rad/s. It is required that the reference spacecraft also rotates about the vertical axis in synchronism with the formation rotation and that the remaining two spacecraft track the attitude of the reference spacecraft. At $t = 100$ s, the spacecraft triad expands to a larger triangular formation with $\mathcal{P}(t)$ given by Eq. (10b) with

$$\mathbf{h}_2^o = -8\mathbf{e}_y - 12\mathbf{e}_z, \qquad \mathbf{h}_3^o = 8\mathbf{e}_y - 12\mathbf{e}_z \qquad (51)$$

Finally, at $t = 150$ s, the formation rotation is repeated. Typical simulation results for control law (19) are shown in Fig. 5, which depicts the trajectories of the spacecraft triad in three-dimensional space. The time records for the quaternions and angular velocities are shown in Figs. 6a and 6b, respectively. The time records for the attitude errors and the norms of the position tracking errors for spacecraft 2 and 3 relative to the reference spacecraft are shown in Figs. 7a and 7b, respectively. The time records for the distances between spacecraft 2 and 3 and the reference spacecraft are shown in Fig. 8. Here, the feedback gains K_{1i}, K_{2i}, K_{qi}, and $K_{\omega i}$ have been tuned to achieve rapid response with acceptable overshoot. Their values are given in the Appendix. Simulation studies for the case with the simplified control law (33) with $\mathbf{K}_{1i} = K_{1i}\mathbf{I}$ and $\mathbf{K}_{2i} = K_{2i}\mathbf{I}$ have also been obtained. The results do not differ appreciably from those given here for control law (19). A video depicting the simulation results for various scenarios in synchronized formation rotation is available.[10]

Conclusion

In this paper, control laws for formation rotation of multiple free-flying spacecraft about a given axis and synchronization of individual spacecraft rotation with formation rotation are derived using a simplified dynamic model in the absence of gravitational field and other disturbances. Simulation studies based on a generic spacecraft model showed that the derived control laws are effective for this mode of operation. In the presence of amplitude limited control thrusts, the derived control laws are effective provided that the formation rotation speed is sufficiently low. Various problems associated with the implementation of the control laws for real spacecraft and the effect of gravitational and environmental disturbances on the spacecraft motion are not considered here. They require further study.

Appendix: Parameter Values for Simulation Study

The spacecraft mass (in kilograms) was $M_1 = 20$; $M_2 = M_3 = 10$.

The moment of inertia about the x_i axis (in kilogram-square meters) was $I_{x1} = 0.7290$; $I_{x2} = I_{x3} = 0.3645$.

The moment of inertia about the y_i axis (in kilogram-square meters) was $I_{y1} = 0.54675$; $I_{y2} = I_{y3} = 0.2734$.

The moment of inertia about the z_i axis (in kilogram-square meters) was $I_{z1} = 0.625$; $I_{z2} = I_{z3} = 0.3125$.

The feedback gains were $K_{1i} = 1$ and $K_{2i} = 10$ where $i = 1, 2, 3$, $K_{q1} = 1$, $K_{q2} = K_{q3} = 0.335$, $K_{\omega1} = 5.5$, and $K_{\omega2} = K_{\omega3} = 10$.

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